Type	Forms	What to do
First Order Linear	1. $y' + p(t)y = g(t)$ 2. $P(t)y' + Q(t)y = G(t)$	1. If the form is in the second case, divide both sides by $P(t)$ to make the equation become the first form. 2. Multiply both sides of the equation by an integrating factor, $\mu(t)$. 3. Let LHS of (2) = $(\mu(t)*y)'$ 4. Solve for $\mu(t)$ 5. Let $(\mu(t)*y)'$ = RHS of (2) 6. Solve for y
Separable Equations	1. $M(x) + N(y)y' = 0$ 2. $M(x)dx + N(y)dy = 0$ 3. $M(x)dx = N(y)dy$	1. Separate the terms with x from the terms with y. 2. Integrate both sides. 3. Move all terms with a variable in it to the LHS and all constants to the RHS.
First Order Homogeneous Equations	This is a special type of separable equation, so the form looks the same. A first order differential equation is homogeneous if it can be written in this form: $y' = f(y/x)$	1. Let $v = y/x$. $y = vx$ $y' = v'x + v$ 2. Treat the new equation as a separable equation and solve. 3. At the end, substitute y/x for v.
Exact Equations	$M(x,y) + N(x,y)y' = 0$ $M(x,y)dx + N(x,y)dy = 0$	1. Check if $dyM = dxN$ **If dyM = dxN:** 1. Let $M = dxF$ and $N = dyF$ 2. Let $F = \int M dx$ 2. Solve $N = dxF$ 3. Solve for $C(y)$ **If dyM ≠ dxN:** 1. Multiply both sides of the equation by an integrating factor, μ.

		2. To know if μ is a function of x, see if (dyM - dxN)/N depends on x only. If it does, then μ is a function of x. If (dyM - dxN)/N does not depend on x only, check if (dxN - dyM)/M depends on y only. If it does, then μ is a function of y. **Note:** μ may be a function of some combination of x and y. There's no rule to show what μ is a function of. 3. Do dy(μM) = dx(μN) and solve for μ. 4. Treat the new equation as an exact equation and solve.
Homogeneous Equations With Constant Coefficients	ay'' + by' + cy = 0	Let $y = e^{rt}$. Hence, $y' = re^{rt}$ and $y'' = r^2e^{rt}$ $ar^2e^{rt} + bre^{rt} + ce^{rt} = 0$ $e^{rt}(ar^2 + br + c) = 0$ $ar^2 + br + c = 0$ Using the quadratic formula, we get 3 different cases: **b^2 - 4ac > 0** 1. Here, r1 ≠ r2 and r1, r2 ∈ R 2. $Y1 = e^{r1t}$ and $Y2 = e^{r2t}$ 3. Y = C1Y1 + C2Y2 **b^2 - 4ac = 0** 1. Here, r1 = r2 and r1, r2 ∈ R 2. $Y1 = e^{r1t}$ and $Y2 = te^{r1t}$ 3. Y = C1Y1 + C2Y2 **b^2 - 4ac < 0** 1. Here, r1 ≠ r2 and r1, r2 ∈ C 2. r1 = λ + iu and r2 = λ - iu **Note:** We will be using r1 only. 3. $Y1 = e^{\lambda t}(\cos(ut))$ and $Y2 = e^{\lambda t}(\sin(ut))$ 4. Y = C1Y1 + C2Y2
Reduction of Order	p(t)y'' + q(t)y' + r(t)y = 0 Y1 = f(t)	1. Let Y2 = v(t)*Y1, where v is an unknown function. 2. Plug in Y2 into the original equation

		and solve for v. 3. Solve for Y2.
Euler's Equation	$t^2y'' + \alpha ty' + \beta y = 0$	Let $y = t^r$, where r is an unknown constant $t^2(t^r)'' + \alpha t(t^r)' + \beta(t^r) = 0$ $(t^2)(r)(r-1)(t^{r-2}) + (\alpha)(t)(r)(t^{r-1}) + \beta(t^r) = 0$ $(r)(r-1)(t^r) + (\alpha)(t)(r)(t^r) + \beta(t^r) = 0$ $(t^r)[(r)(r-1) + (\alpha)(t)(r) + \beta] = 0$ $r^2 + (\alpha-1)r + \beta = 0$ Using the quadratic formula, we get 3 different cases: **b^2 - 4ac > 0** 1. Here, $r1 \neq r2$ and $r1, r2 \in R$ 2. $Y1 = t^{r1}$ and $Y2 = t^{r2}$ 3. Y = C1Y1 + C2Y2 **b^2 - 4ac = 0** 1. Here, r1 = r2 and $r1, r2 \in R$ 2. $Y1 = t^{r1}$ and $Y2 = \ln(t)t^{r1}$ 3. Y = C1Y1 + C2Y2 **b^2 - 4ac < 0** 1. Here, $r1 \neq r2$ and $r1, r2 \in C$ 2. $r1 = \lambda + iu$ and $r2 = \lambda - iu$ **Note:** We will be using r1 only. 3. $Y1 = t^{\lambda}(\cos(u*\ln(t)))$ and $Y2 = t^{\lambda}(\sin(u*\ln(t)))$ 4. Y = C1Y1 + C2Y2
Non-Homogeneous Equation	$y'' + p(t)y' + q(t)y = g(t)$	Too many possibilities